CHAPTER 2

THEORY

This chapter will discuss the theoretical background for this analysis. The SM is summarized in Section 2.1, and the relevant aspects of the SM EFT framework are discussed in Section 2.2.

2.1 The standard model

The standard model (SM) of particle physics is the mathematical framework that describes fundamental particles and their interactions. The SM is a quantum field theory with $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_Y \times \mathrm{U}(1)_w$ gauge symmetry. The $\mathrm{SU}(3)_c$ component corresponds to QCD, and is non-abelian. The $\mathrm{SU}(2)_w$ component is referred to as weak isospin, and is also non-abelian. The $\mathrm{U}(1)_Y$ group is referred to as hypercharge, and it is abelian. The SM Lagrangian contains kinetic terms for each of these three gauge fields:

$$\mathcal{L}_{SM} \supset -\frac{1}{4}(G^A_{\mu\nu})^2 - \frac{1}{4}(W^a_{\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2, \tag{2.1}$$

where $G^A_{\mu\nu}$ is the $\mathrm{SU}(3)_c$ field strength tensor (with $A = 1...8$), $W^a_{\mu\nu}$ is the $\mathrm{SU}(2)_w$ field strength tensor (with $a = 1...3$), and $B_{\mu\nu}$ is the $\mathrm{U}(1)_Y$ field strength tensor.

In the SM, the $\mathrm{SU}(3)_c$ symmetry is exact, while the $\mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ is spontaneously broken by the Higgs mechanism. In order to preserve $\mathrm{SU}(3)_c$, the Higgs field ϕ must transform as a singlet under $\mathrm{SU}(3)_c$; in order to break $\mathrm{SU}(2)_w$ and $\mathrm{U}(1)_Y$, the Higgs field must be charged under these symmetries. The Higgs field is a doublet under $\mathrm{SU}(2)_w$, and has a hypercharge of 1/2. This can be expressed as $\phi = (0, 2, 1/2)$, where the first number corresponds to the $\mathrm{SU}(3)_c$ representation, the second number corresponds to the $\mathrm{SU}(2)_w$ representation, and the third number corresponds to the $\mathrm{U}(1)_Y$ representation. With the inclusion of the Higgs field, the SM Lagrangian contains:

$$\mathcal{L}_{SM} \supset |D_\mu \phi|^2 + V(\phi), \tag{2.2}$$

where $D_\mu \phi$ is the covariant derivative and V is the potential, given by

$$V(\phi) = \lambda \left(|\phi|^2 - \frac{v^2}{2} \right)^2, \tag{2.3}$$

which is minimized when $|\phi|^2 = v^2/2$. The Higgs field ϕ is a complex doublet:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 + ih_2 \\ h_0 + ih_3 \end{pmatrix}, \tag{2.4}$$

so we have $\phi^\dagger \phi = 1/2\ (h_0^2 + h_1^2 + h_2^2 + h_3^2)$. We know that $V(\phi)$ is minimized when $\phi^\dagger \phi = v^2/2$, but there are infinitely many ways to satisfy this. We chose $\langle h_0 \rangle = v$, with $\langle h_1 \rangle = \langle h_2 \rangle = \langle h_3 \rangle = 0$, breaking the symmetry. With this choice, the vacuum expectation value for ϕ is:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2.5}$$

Plugging equation 2.5 into the Higgs kinetic term $|D_\mu \phi|^2$ gives rise to the mass terms for the massive gauge bosons. Expanding Eq. 2.5 around the minimum, we have

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \tag{2.6}$$

where h is the physical Higgs boson particle.

Next, let us consider the matter fields in the SM. The matter fields and their $\mathrm{SU}(3)_c \times \mathrm{SU}(2) \times \mathrm{U}(1)$ representations are listed in Table 2.1.

TABLE 2.1

MATTER FIELDS IN THE SM.

	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_w$	$\mathrm{U}(1)_Y$
Q	3	2	1/6
u_R	3	0	2/3
d_R	3	0	-1/3
L	0	2	-1/2
e_R	0	0	-1

In Table 2.1, the left-handed quark doublet is written as Q; the components of this $\mathrm{SU}(2)_w$ doublet can be written as u_L and d_L. The right-handed quark singlets are written as u_R and d_R. The u quarks are referred to as up-type quarks, while the d quarks are referred to as down-type quarks. The left-handed lepton doublet is written as L, with $\mathrm{SU}(2)_w$ components e_L (which is electrically charged) and ν_L (which is electrically neutral). The right-handed charged lepton is written as e_R. There is no known right-handed neutral lepton.

There are three generations of each of the matter fields listed in Table 2.1. The generations of up-type quarks are referred to as up (u), charm (c), and top (t), while the three generations of down-type quarks are referred to as down (d), strange (s), and bottom (b). The three generations of charged leptons are referred to as electrons (e), muons (μ), and taus (τ), while the three generations of neutral leptons are referred to as electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ).

We can write kinetic terms (of the form $i\bar{\psi}\not{D}\psi$) for each of the fermion fields. Since these terms involve either two left-handed fields or two right-handed fields, the kinetic terms are gauge invariant. However, because the left and right hand fields transform differently under $\mathrm{SU}(2)_Y \times \mathrm{U}(1)_w$ (i.e. the left and right hand fields have different $\mathrm{SU}(2) \times \mathrm{U}(1)$ representations), we cannot write Dirac mass terms for the matter fields, since these terms would not be invariant under the SM symmetries. We can, however, use the Higgs field to write Yukawa terms for the matter fields, which gives rise to mass terms and to terms that describe the interactions between the fermions and the Higgs boson.

Putting together Eq. 2.1 (the kinetic terms for the gauge fields), Eq. 2.2 (the kinetic and potential terms for the Higgs field), the kinetic terms for the fermion fields, and the Yukawa terms for the fermion fields, we obtain the full SM Lagrangian. The SM Lagrangian is written as $\mathcal{L}_{\mathrm{SM}}$ in Eq. 1.1.

2.2 Effective field theory

As introduced in Chapter 1, EFT provides a general framework for describing the off-shell effects of heavy new physics as an expansion of higher-dimensional[1] operators. The operators are constructed of products of SM fields and their derivatives. At each order in the expansion, the operators are scaled by powers of Λ, the mass scale of the new physics. All operators of odd dimension violate baryon and/or lepton number [11], so are not considered in this analysis. Operators of dimension six thus represent the leading new physics effects and are the focus of this analysis.

[1]Here dimension refers to the mass dimension of the operator in natural units (where $\hbar = c = 1$ and all units are expressed as energy to some power). The SM operators are of dimension four, so "higher dimensional" refers to operators of dimension greater than four.

The dimension-six operators can be expressed in different bases, the most common of which is known as the Warsaw basis [12]. The number of operators at each dimension depends on the flavor symmetries that are assumed. Under the assumption that all three generations may vary independently, there are 2499 operators at dimension six; assuming flavor universality, this number reduces to 59 (assuming lepton number and baryon number conservation) [13].

Adopting a flavor symmetry assumption in between the two extremes, the model presented in Ref. [14] is the EFT model used in this thesis. This model is referred to as the dim6top model; it makes use of the Warsaw basis and provides tree-level modeling for dimension-six operators. Developed to facilitate studies focusing on third-generation effects, the dim6top model has compiled the set of 33 dimension-six operators involving two or more third-generation quarks. In the dim6top model, the operators are assumed to be invariant under $\mathrm{U(2)_Q \times U(2)_u \times U(2)_d}$, so the couplings for operators involving third generation quarks may vary independently from the first two generations. While dim6top allows for EFT effects to vary independently for each generation of leptons, this analysis imposes the assumption that the EFT effects impact each lepton generation in the same way.

In this analysis, we aim to include all operators from the dim6top model that significantly impact processes in which one or more top quarks are produced in association with charged leptons; as listed in Table 2.2, this comes to 26 operators in total. The definitions of the WCs in Table 2.2 and the definitions of the corresponding operators can be found in Table 1 of Ref. [14]. However, for the vertices involving the c_{tG} WC, there is one important difference with respect to the definition in Ref. [14]. In order to allow MadGraph to properly handle the emission of gluons from the vertices involving the c_{tG} WC, an extra factor of the strong coupling is applied to the c_{tG} coefficients (as explained in [15]).

TABLE 2.2

LIST OF WILSON COEFFICIENTS INCLUDED IN THE ANALYSIS.

Category	WCs
Four heavy	c_{QQ}^{1}, c_{Qt}^{1}, c_{Qt}^{8}, c_{tt}^{1}
Two light two heavy	c_{Qq}^{31}, c_{Qq}^{38}, c_{Qq}^{11}, c_{Qq}^{18}, c_{tq}^{1}, c_{tq}^{8}
Two heavy two lepton	$c_{Q\ell}^{3(\ell)}$, $c_{Q\ell}^{-(\ell)}$, $c_{Qe}^{(\ell)}$, $c_{t\ell}^{(\ell)}$, $c_{te}^{(\ell)}$, $c_{t}^{S(\ell)}$, $c_{t}^{T(\ell)}$
Two heavy with bosons	$c_{t\varphi}$, $c_{\varphi Q}^{-}$, $c_{\varphi Q}^{3}$, $c_{\varphi t}$, $c_{\varphi tb}$, c_{tW}, c_{tZ}, c_{bW}, c_{tG}

The 26 operators fall into four main categories: operators involving 4 heavy quarks, operators involving two heavy quarks and two light quarks, operators involving two heavy quarks and two leptons, and operators involving two heavy quarks and bosons. The vertices arising from these operators can impact the signal processes of this analysis, as illustrated in the example diagrams in Figure 4.2. The four four-heavy WCs only have significant impacts on the $\mathrm{t\bar{t}t\bar{t}}$ signal process, so these WCs are not included in the modeling of the other five signal processes. The details of the Monte Carlo generation of the signal samples will be discussed in Section 3.2.1.

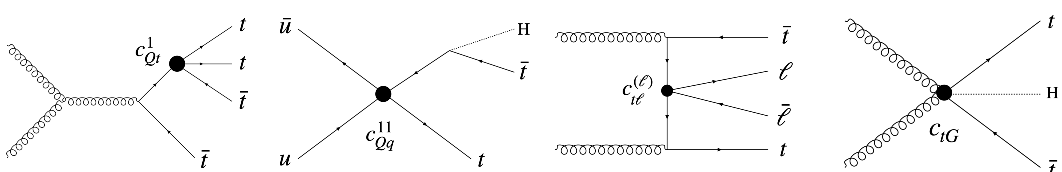

Figure 2.1. Example Feynman diagrams illustrating WCs from each of the categories listed in Table 2.2. From left to right, the diagrams show vertices associated with the c_{Qt}^{1}, c_{Qq}^{11}, $c_{t\ell}^{(\ell)}$, and c_{tG} WCs.

For each of the six signal processes, we account for diagrams with zero EFT vertices (i.e. the SM contribution) and diagrams with one EFT vertex (i.e. the new physics contribution). The amplitude for each process will thus depend linearly on the WCs, and the cross sections will depend quadratically on the WCs. Since the weight of each generated event corresponds to the event's contribution to the inclusive cross section, each event weight will also depend quadratically on the 26 WCs; the parametrization of the event weights in terms of the WCs is the key concept that allows us to obtain detector-level predictions in terms of the WCs and will be discussed in detail in Section 3.2.2.